INDE 301
Engineering Economy

Sensitivity Analysis

Dr. N. Nehme

LEARNING OBJECTIVES

1. **Explain sensitivity to parameter variation**
2. **Use three estimates for sensitivity analysis**
3. **Calculate expected value E(X)**
4. **Determine E(X) of cash flow series**

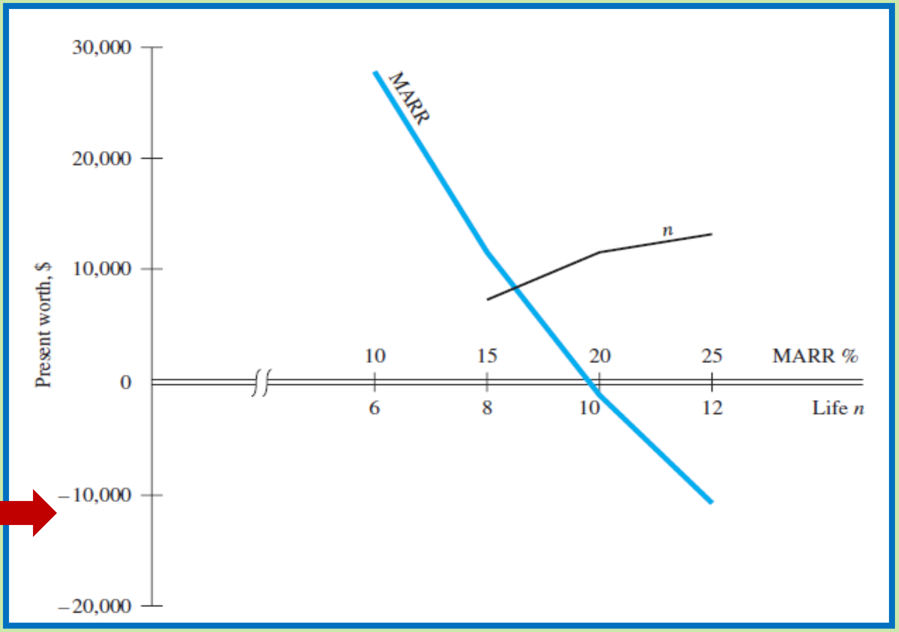
Parameters and Sensitivity Analysis

- **Parameter** -- A variable or factor for which an **estimated or stated value** is necessary

- **Sensitivity analysis** – An analysis to determine **how a measure of worth** (e.g., PW, AW, ROR, B/C) **changes** when one or more parameters vary over a **selected range of values.**

PROCEDURE:

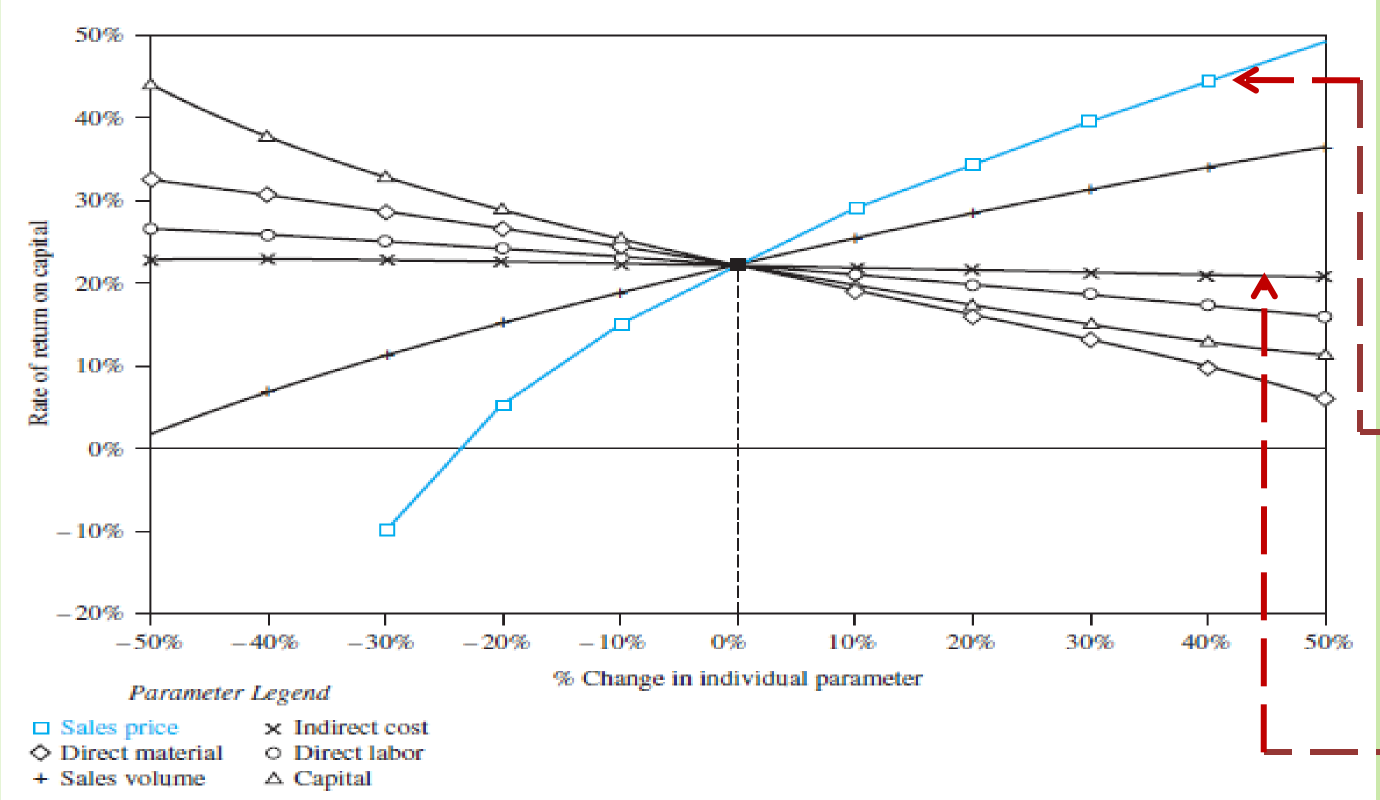
1. Select **parameter** to analyze. Assume **independence** with other parameters
2. Select probable **range and increment**
3. Select **measure of worth**
4. **Calculate measure** of worth values
5. **Interpret results.** Graph measure vs. parameter for better understanding



Sensitivity of Several Parameters

When several parameters for one alternative vary and analysis of each parameter is required …

graph percentage change from the most likely estimate for each parameter **vs. measure of worth**



Plots with larger slopes (positive or negative) have a higher sensitivity with parameter variation (sales price curve)

Plots that are relatively flat have little sensitivity to parameter variation (indirect cost curve)

 Dr. N. Nehme

Three Estimate Sensitivity Analysis

- Applied when selecting one ME alternative from two or more
- For each parameter that warrants analysis, provide **three estimates**:
  - Pessimistic estimate ***P***
  - Most likely estimate ***ML***
  - Optimistic estimate ***O***
- Calculate measure of worth for each alternative and 3 estimates and select 'best' alternative

Notes -- 1. The pessimistic estimate may be the **lowest** for some parameters and the **highest** for others, e.g., low life estimates and high first cost estimates are usually pessimistic

2. When calculating the measure of worth, **use ML estimate** of a parameter as others varies. This is the **independence** assumption

Expected Value Calculations

Expected Value -- Long-run average observable if a project or activity is repeated many times

Result is a point estimate based on anticipated outcomes and estimated probabilities

$$E(X) = \sum_{i=1}^{m} X_i P(X_i)$$

Where: X_i = value of variable X for i = 1, …, m different values

$P(X_i)$ = probability that a specific value of X will occur

In all probability statements, the sum is:

$$\sum_{i=1}^{m} P(X_i) = 1.0$$

When E(X) < 0, e.g., E(PW) = $-2550, a ***cash outflow*** is expected; the project is ***not*** expected to return the MARR used

Dr. N. Nehme

Example: Probability and Expected Value

Monthly M&O cost records over a 4-year period are shown in \$200 ranges. Determine the expected monthly cost for next year, if conditions remain constant.

Range,\$, X	No. of months	Range,\$, X	No. of months
100 - 300	4	700 - 900	6
300 - 500	12	900 - 1100	10
500 - 700	14	1100 - 1300	2

Solution: P(X) = number of months / 48 months

$$E(X) = 200(4/48) + 400(12/48) + \cdots + 1200(2/48)$$

$$= 1/48[200 \times 4 + 400 \times 12 + \cdots + 1200 \times 2]$$

$$= 1/48[31{,}200]$$

$$= \$650 / \text{month}$$

Expected Value for Alternative Evaluation

Two applications for Expected Value for estimates:

1. Prepare information for use in an economic analysis
2. Evaluate economic viability of fully formulated alternative

Example: Second use for a complete alternative. ***Is the investment viable?***

	Economic Condition		
	Receding (Prob. = 0.4)	Stable (Prob. = 0.4)	Expanding (Prob. = 0.2)
Year	Annual Cash Flow Estimates, $		
0	−5000	−5000	−5000
1	+2500	+2500	+2000
2	+2000	+2500	+3000
3	+1000	+2500	+3500

P = $-5000

n = 3 years

MARR = 15%

Example: Expected Value for Alternative Evaluation

	Economic Condition		
	Receding (Prob. = 0.4)	Stable (Prob. = 0.4)	Expanding (Prob. = 0.2)
Year	Annual Cash Flow Estimates, $		
0	−5000	−5000	−5000
1	+2500	+2500	+2000
2	+2000	+2500	+3000
3	+1000	+2500	+3500

Solution: **Calculate PW value for each condition**

PW_R = -5000 + 2500(P/F,15%,1) + 2000(P/F,15%,2) + 1000(P/F,15%,3)
= **$-656** (cash outflow; not viable)

PW_S = $+708 (cash inflow; viable)

PW_E = $+1309 (cash inflow; viable)

Now, calculate expected value of PW estimates

$E(PW) = PW_R \times P(R) + PW_S \times P(S) + PW_E \times P(E)$

$= -656 \times 0.4 + 708 \times 0.4 + 1309 \times 0.2$

= $+283

On basis of E(PW) > 0 at 15% over 3 years, investment is viable

